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ARTIFICIAL LANGUAGES ACROSS SCIENCES AND CIVILIZATIONS

anupāsitavṛddhānāṃ vidyā nātiprasīdati
"science does not smile on those who neglect the ancients"
Bhartṛhari, *Vākyapadīya*

INTRODUCTION

Beneath this essay lies a philosophic question: how can language, natural or artificial, assist us in knowledge of the world (which includes ourselves)? I shall not try to contribute to the answer of this question by mere speculation, but start with some facts: artificial languages are at least as universal as natural languages and are neither restricted to one civilization, viz., the so-called Western which I call Euro-American; nor are they confined to mathematics, physics and a few other sciences that in English are often referred to as 'exact.'

My paper consists of four parts. Parts I and II pave the way for III and IV. Part I deals with ancient and medieval science. Its theatre is Asia, the large continent to which Europe is appended. Part II sketches the sciences of language that developed there and provides a background for the emergence of artificial languages that will be-treated in Parts III, 'Early Artificial Languages' and IV, 'From Natural to Artificial Language.'

Definitions come in the end, but the concept of artificial language that is used here must be clarified at least to some extent. Artificial languages contrast with natural languages such as English or Thai. An artificial language is not natural – or so it seems. But the opposite does not hold. Natural languages may be artificial and artificiality admits of degrees. In English, it is natural to say: *the roof of the house.* To say: *the roof the house has* seems to mean the same but some speakers may feel that it is slightly less natural or more artificial.

Similarly, *the chimney on the roof of the house* is natural. But **the chimney the roof the house has has* is artificial in the sense that it is not English (as the asterisk * indicates); and yet it is in accordance with the same rules of embedding that are part of English syntax. According to George Miller, a few Harvard undergraduates could in 1964 do two such embeddings, but everyone had trouble with three or more. Facts like these show that some of the rules of an artificial language are the same as, or are similar to some of the rules of a natural language.

Artificial languages are not the same as formal languages. Is a formal language always artificial but an artificial language not always formal? I put these questions to Sol Feferman and asked him to explain to me the difference between the two and especially the artificial variety. Professor Feferman said, basically, I know a formal language when I see one and added something about systematic, compositional and symbolic. On artificial languages he referred me to three websites that are listed at the end of the References list. He drew my particular attention to the definition that is given in the third of these and that is due to Rick Harrison:

> An artificial language is a language that has been deliberately designed by one person or a small group of people over a relatively short period of time. Synonyms for the term artificial language include planned language, constructed language, model language, and invented language. Artificial languages designed for specific purposes are also known by an array of other terms. Those used in works of fiction are called imaginary languages or fictional languages. Those designed to facilitate global communications are called universal languages, auxiliary languages (auxlangs), interlanguages or interlinguas, international languages, etc. The realm of artificial languages also includes logical languages, number languages, symbolic languages, and pasimologies (gesture languages).

The artificial languages with which we shall be concerned in this essay are tucked away in the final phrase of this enumeration. They have been designed for use in sciences such as mathematics, logic, linguistics, physics or computer science. They are often formal. Even classical languages of science such as Latin or Sanskrit may be artificial or formal to some extent. Most of these languages may be studied for their own sake, without concern for their meaning or semantic interpretation. They are invariably compositional or syntactic in that they possess syntax, a mechanism that enables them to construct compound expressions from simple ones in a manner that is analogous to the construction of sentences (or other compound expressions) from words (or other elementary expressions) in a natural language.

The most creative and powerful artificial languages are the languages of mathematics. History shows that they have *not* been 'deliberately designed by one person or a small group of people over a relatively short period of time.' On the contrary, these languages grew very, very slowly. Some of their expressions took more than three thousand years to arrive at their present forms which are, of course, not their last. The modern symbol for negation, '–', for example, started with a cuneiform expression in Old-Babylonian. The Indians put a dot above a negative number, but the symmetries between positive and negative and between addition and subtraction were only expressed in a manner that made calculation wieldy when '+' and '–' came into use in the fifteenth century CE (Tropfke, 1980 : I, 144–146, 206). The development of expressions for equations took equally long but was much more complex (Tropfke, 1980: I, 382 ff.).

The language of mathematics is sometimes regarded as if it were constant and invariable, a necessary reflection of the universe, a metaphysical symbolism almost divine. An unnamed writer in *The Economist* (December 20, 2003) declared that Latin was for Newton 'the closest approach in words to the utter directness of mathematical symbols.' That does not only ignore history but puts it on its head. It took almost a century before Euler, Daniel Bernouilli and others translated Newton's laws from Latin into equations, making use of Asian algebras that were streaming into Europe via the Arabs (Staal, 1995a: 75–76, 2004 and below). Leibniz went a step further and declared that progress in mathematics is largely due to improved notations (Mates, 1986, Ch. X), that is, to the development of artificial expressions and languages.

I ANCIENT AND MEDIEVAL SCIENCE

1. *The Unity of Pre-Modern Science*

'Science' is often described or imagined as 'Western', having originated in ancient Greece with the Arabs acting as translators. It is a prejudice of long standing and derives from an outdated picture of the history of science. The history of pre-modern, that is: ancient and medieval science can only be adequately understood if the Eurasian continent is treated as an undivided unit. That insight evolved over more than half a century, roughly speaking from Otto Neugebauer and Joseph Needham to David Pingree, and is based upon the textual and historical study of source materials in the classical languages of science that include Arabic, Old-Babylonian, Chinese, Greek, Latin and Sanskrit.

The accompanying chart (Figure 1) portrays some of the historical relationships between these civilizations. It is one of a large number of possible simplifications. My picture omits languages and entire civilizations, mixes geographic appellations with language names and emphasizes astronomy and mathematics. Even so, a few things are clear. Mesopotamian science is earlier than any other and has exerted some influence, directly or indirectly, on China, Greece and India. Greek and Indian geometries seem to be related though little is known of or agreed upon historical channels.[1] What is obvious is that the Arabs stood at the geographical and historical center of pre-modern science.

Quotation marks in the chart draw attention to simplifications that may be especially misleading. 'Hellenistic' is placed between quotes because Neugebauer, who was concerned to show that not all Hellenistic science was Greek, continued to use the term which perpetuates the view that, in fact, it was – an erroneous view if only because algebra, a pivotal-topic in our present context, did not come from the ancient Greeks. Quotation marks surround 'Arab' because Arab science did not arise from the Arab peninsula though it was generally expressed through the Arabic language. 'Arab' science is also referred to as 'Islamic', but since Jews and Christians contributed to it, that appellation is equally misleading.

The label that sounds especially pretentious is *Modern*. I have not put it between quotes because I believe it is correctly used. But *modern* is not the same as *Western* or *Occidental*, terms that lead as much astray as their counterparts *Eastern* or *Oriental* that are now considered incorrect. Modernization is inherent in any form of progress but the term is used when the event has just occurred. The *modern* of 'modern science' refers to the most recent event of radical progress in the realm of knowledge. But in order to be able to determine how radical it was, we need to adopt a wide perspective.

[1] I was wrong in claiming that the Vedo-Greek variety of geometry is different from all Mesopotamian mathematics (Staal, 1999: 107) as Jens Høyrup has explained to me. See Høyrup (1998: 32–40) which shows that *Elements* II.1–10 largely consists of 'cut -and-paste' results long known from Old-Babylonian sources. I like to use this opportunity to clarify my statement, that Brahmagupta's expressions suggest the notion of an equation (Staal, 1995a: 91). Suggestive they may be, but Hayashi (1995: 92, 333) has shown that they are not equations but tabular presentations of the numerical data from which solutions are obtained with the help of an algorithm.

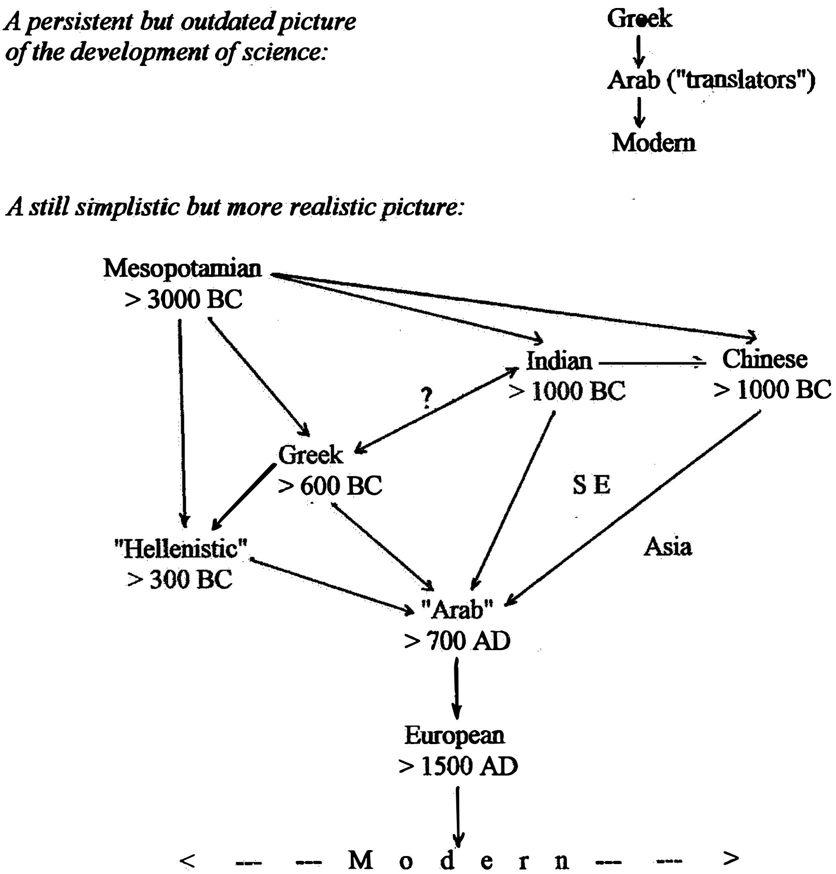


Figure 1. The History of Ancient and Medieval Science.

Modern science started with the scientific revolution which occurred in certain sciences and in Europe though it is not fashionable to refer to it as such. Many academics avoid the expression and it is possible to ignore it when one's gaze is confined to Europe. If one looks beyond that subcontinent, it is obvious that some such event occurred there during that time. It explains that Joseph Needham, whose work is not only about China but abounds in references to Indian, Near-Eastern and European sciences, was obsessed by the question why the scientific revolution occurred in Europe and not in China or India. Whatever it is, we shall see that the subsequent explosion of mathematical activity through artificial languages during the seventeenth and eighteenth centuries in Europe was perhaps the greater revolution, and that is not the end of the story.

The arrows of Figure 1 do not only claim that the ancient and medieval science of the Eurasian continent *preceded* modern science but *led* to it. That is an empirical hypothesis that we shall subject to closer scrutiny in Section 4 'Rivers Flowing into the Sea: An Empirical Hypothesis'. Whatever its empirical content, any such hypothesis depends on non-empirical, *a priori* principles as well. One of these is that an ancient problem cannot be understood unless subsequent study has thrown more light upon the matter. Joseph Needham put his finger on it when he wrote: 'to write the history of science we have to take modern science as our yardstick – that is the only thing we can do – but modern science will change, and the end is not yet' (Needham, 1976: xxxi). I refer to this yardstick principle as the principle of modernity and since it comes first in terms of methodology, is widely misunderstood and plays a central role in the following discussions, I shall examine it in Section 3 'The Principle of Modernity'.

Needham's final clause – that science is never final – applies to the history of science itself. Assume that we were to discover that the number system of the Mayans, which developed during the first millennium CE, derives from an ancient American civilization that also influenced Asia. Would it not affect our ideas about the early history of mathematics? It is an unlikely possibility in the light of what is presently known, but what will we know to-morrow?

Meso-American cultures contrast instructively with our 'Eurasian' picture because they grew in total isolation until the sixteenth century. No one has expressed it more clearly than Octavio Paz, Mexico's great poet and her ambassador to India:

> India was always in communication with other peoples and cultures of the Old World: first with Mesopotamia, and later with the Persians, Greeks, Kuchans, Romans, Chinese, Afghans, Mongols. The thought, religions and art of India were adopted by many Asian peoples; in turn, the Indians absorbed and transformed the ideas and creations of other cultures. The Mexican peoples did not experience anything like (that)... They lived in an immense historical solitude; they never knew the essential and common experience of the Old World: the presence of the Other, the intrusions of strange civilizations with their gods, technical skills, visions of this world and the next (Paz, 1995: 91).

Multiculturalism, the correct fashion especially in the USA, treats Arabs, Chinese, Euro-Americans and Indians as if they inhabited separate cognitive worlds and were as isolated as were the Meso-American cultures. That idea is not supported by the history of science.

2. *The Convergence of East and West: c 1200–1400 CE*

Why do the Arabs stand at the geographical and historical center of pre-modern science? Part of the answer is given by Figure 2 (Schwartzberg, 1978: 37) which takes us from history to geography and from science to military campaigns and conquests. The region in the middle, surrounded by a thick black line, like the larger overlapping area to its east, is the boundary of the empire of Timur or Tamerlan around 1400 CE. It includes most of ancient Mesopotamia and a good part of the area of Hellenism but its center of gravity lies further east. It was then and under Mongol rule that freedom of travel and social intercourse were least restricted and hospitality to strangers reached its highest peak (Schwartzberg, 1978: 196–197).

The map of Figure 2 shows that anything like the so-called scientific revolution could not have taken place before, say, 1500 CE. Prior to the twelfth century, many of the areas on the map were much more advanced than Europe and we could, with hindsight, expect them to have engendered a scientific revolution. Actually they did, as we shall see, but in the sciences of language, areas to which most historians of science do not pay much attention. In the mathematical and physical sciences, the revolution happened later and further to the west. By the twelfth and thirteenth century, 'Islamic and Christian centuries were growing on roughly the same level and it is then sometimes hard to know in which society a new development came first' (Hodgson, 1974, II: 365). 1 conclude that *it is not the place that was decisive but the time.*

3. *The Principle of Modernity*

Like other sciences, the history of science itself does not only depend on empirical facts but also on *a priori* principles. Conversely, principles may depend on facts from the history of science. That circularity or apparent circularity is in accordance with the absence of principled boundaries between facts and theories that philosophers like W.V.O. Quine and others have elucidated. No such circularity implies that there are no facts at all and that the relativism that denies their existence makes any sense. These problems are relevant to our discussion but this is not the place to discuss them in any detail.[2]

[2] Modern philosophers of science do not often take account of the history of science, let alone its non-European branches. Ian Hacking writes in the introduction to his readings about scientific revolutions: 'Newton's term 'mass' may not even

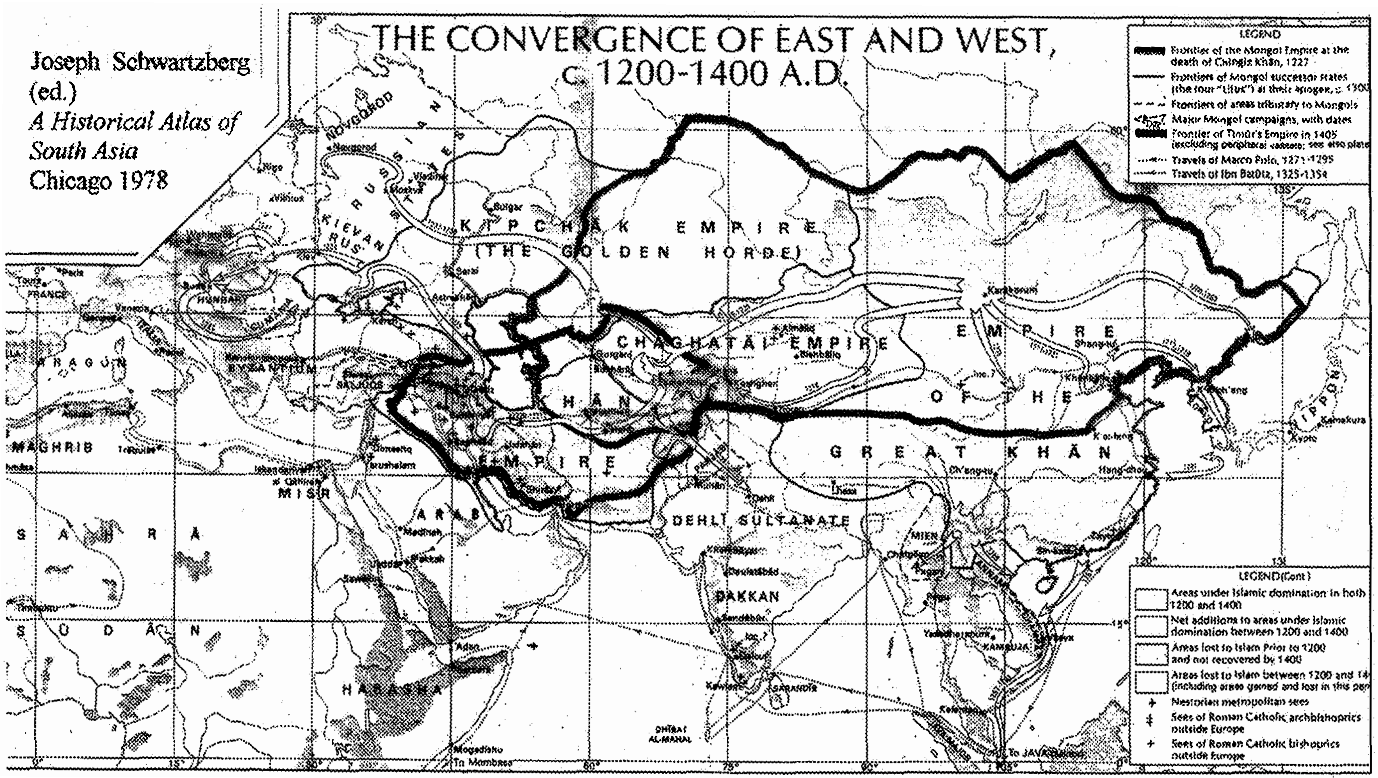


Figure 2. The Covergence of East and West: c. 1200–1400 CE.

I shall briefly return to relativism in connection with mathematics (in the present section) and to another principle that is methodological or epistemological (in Section 5 'The Principles of Acceptance'); but the principle of modernity that will be considered here is different. Needham had a particularly clear-headed understanding of it and I have already quoted the way in which he formulated it: 'to write the history of science we have to take modern science as our yardstick – that is the only thing we can do – but modern science will change, and the end is not yet.'

Specialists in the history of Chinese science often criticize or even ignore Needham. I cannot judge these discussions but some of the critics seem to have already forgotten that Needham was a pioneer who brought together 'the raw material on which generations of later scholarship can be founded' (de Solla Price, 1971: 17–18). Many historians of science are similarly unhappy with regard to the yardstick/modernity principle. I discussed some of these issues before (Staal, 1993–1994: 9–19) but will to take a fresh look at some recent publications which are useful because they exemplify not what the principle is, but what it is not.

G.E.R. Lloyd

G.E.R. Lloyd's recent comparison between ancient Greece and China is learned and sophisticated and he is engaging writer. His work is published in a distinguished Cambridge series, 'Ideas in Context', which seeks to dissolve 'artificial distinctions between the history of philosophy, of the various sciences, of society and politics, and of literature'. Artificial things should be dissolved if it can be done. It does not apply to artificial language itself despite the belief of some so-called ordinary-language philosophers. But let me not digress. The title of Lloyd's third chapter is 'The number of things'. It begins with the following paragraph which throws

Footnote 2 continued.
mean what it does in Einstein's relativistic physics' (Harkin, 1981: 3) Such insights are obvious to historians of science who study, among other things, changes in the meaning of basic concepts, especially across language boundaries. As for relativism, discussions continue unabated though it refutes itself: for any argument in support of relativism must assume that the argument is relative and therefore not an argument. See, e.g., Scharfstein, Staal and others in Ariel *et al.* (1998) and Scharfstein (2001).

light not only on our present discussion but on many topics that will engage our attention later:

> From the belief that mathematics reveals invariant truths it is all too tempting to conclude that mathematics itself is invariant. That view is then often combined with a thesis about the development of science, namely that it depends on a shift from the qualitative explanation of phenomena to their quantitative understanding. That is generally represented as the key factor in the changes that took place in what used to be called the scientific revolution of the sixteenth and seventeenth centuries, and in support Galileo is regularly cited for the notion that the book of the universe is written in mathematical language (Llyod, 2002: 44).

Llyod continues: 'The flaws in such a package of beliefs are obvious'. The first of these flaws, according to him, is writing history 'from the perspective of what was to come'. This may or may not be a veiled attack on Needham but is different from the yardstick/modernity principle. Llyod refers to earlier chapters of his book to support his statement. What these chapters do, and what Llyod continues to do, is stress the immense varieties we come across in history. He does that with great verve and is fully convincing.

As I see it, there are four separate strands in the paragraph I have quoted.

α. The first is the relativistic stance that appears also elsewhere in the book. It is particularly unpersuasive in the case of number and mathematics in general. Mathematics does not seek to dissolve 'artificial distinctions between the history of philosophy, of the various sciences, of society and politics, and of literature'. It *abstracts from context*, by definition. It does not study 'two apples' or 'two mangoes' but *two.* It is therefore by nature recalcitrant to context which, in turn, does not throw any light on its nature. But does Lloyd himself accept that mathematical truths are relative?

I don't believe that Lloyd believes that $2 \times 3 = 6$ is flawed, varies over time or that we may accept with equal equanimity that $2 \times 3 = 5$. If that were the case, he would experience continuous problems with his fellow human beings, not to mention his calendar, plans or recollections, and many other issues in his life. Without knowledge of such properties of numbers, ancient Chinese and Greeks would have had the same problems, not to mention great difficulty in operating their various types of counting-board. Nor is such invariance confined to elementary arithmetic. '317 is a prime, not because we think so, or because our minds are shaped in one way or another, but *because it is so*' (Hardy, 1994, reprint: 130).

Lloyd has a great way with words and is fond of looking at them. He is inclined to restrict language to words without taking into account that it consists of sentences and other compound structures. In his chapter on 'The language of learning', he confines himself to vocabulary and terminology, that is not merely words but mostly nouns. In the chapter on number, he is concerned to show that Chinese *shu* covers a much wider range than Greek *arithmos*. It may mean, for example, 'several', 'counting' and 'scolding'. Such uses occur in many languages. Greek itself says ἐν ἀριθμῷ εἶναι 'being counted' that is 'being honored', as well as ἐν οὐδενὶ ἀριθμῷ εἶναι 'being despised'. English says: 'he made a large number of mistakes'. Not only in mathematics itself, but even in a discussion of mathematics, the words of a natural language do not count for much – and yes, I accept that that applies to my words, too.

β. Lloyd is admirably successful when he shows that the shift from qualitative to quantitative explanation is an ancient one that occurred in early China (as it did in India) and did not have to wait for Galileo. It occurred in a variety of contexts and it is in the treatment of these that Lloyd is at his best. And yet, later in the chapter, in the longest sustained discussion of the contextual use of a mathematical figure, viz, the gnomon (pp. 52–55), Lloyd himself frees himself from contexts and his self-imposed relativism both. He concludes with reference of calculations from gnomon shades, that the key difference between Greece and China lies 'in the starting assumption made' and adds that 'Chinese and Greeks methods were essentially the same, mathematically'.

γ. The next two strands present us with a puzzle. It is clear that the flawed considerations of *β*. imply that the shift from qualitative to quantitative cannot be the key factor in the changes in 'what used to be called the scientific revolution of the sixteenth and seventeenth centuries'. We may accept the tiptoeing around that revolution as a small bow to correct behavior, but does Lloyd deny that something happened during that period? That would be odd, for although that error is readily made by those who do not look beyond Europe, it is an event that could not be missed by anyone familiar with developments in other civilizations during that period. Lloyd need not be obsessed by it, as was Needham, and it is true that Lloyd is primarily interested in ancient texts. But why then does he venture here into those later centuries?

δ. Deepens mystery because it involves Galileo who did write what is always quoted in part or in full, viz.: 'Philosophy is written in this grand book – I mean the universe – which stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language and interpret the characters which it is written. It is written in the language of mathematics . . .' (quoted after Blay, 1993/1998: 1). Lloyd has shown that this does not refer to the transition from quantitative to qualitative. We shall see later what it does refer to.

Elsewhere in his book, Lloyd pays due attention to the Euclidian tradition of axiomatic-deductive demonstration which, in earlier publications (Lloyd, 1990, 1996 a and b), he had discussed in relation to Aristotle's and other philosophers' essays on demonstration as well as the practice and theory of Greek rhetoricians and orators. He sees these related uses as a proof that the Euclidian tradition is not merely 'intellectually attractive'. Here his relativist colors begin to shine more brightly – but what, to a relativist, is a proof? Lloyd confines himself to two of the three levels that should be taken into account:

The first level is that of Euclid. It is concerned with mathematics. It *proves*, for example, that the theorem of Pythagoras follows from the axioms. If Lloyd does not accept that proof, 'intellectually attractive' as it is, he should at least show that it does not so follow which would earn him the Field Medal or Schock Prize, equivalents to the Nobel.

The second level is that of physics. Euclid did not know physics – or did he? We accept his parallel postulate because it describes a flat surface. If we reject it, we can still use the rest of the system to describe a curved one. Not just Euclid's system, but geometry and mathematics in general, are known to apply to the universe, i.e., they may be *true*. Lloyd is silent about it.

The third level is that of the derivative uses of such systems in debates, rhetoric and oratory. Lloyd revels in it.

Are my first and second levels 'writing history from the perspective of what was to come'? No, they show something else. Euclid's demonstration is from axioms which, following Aristotle, he called 'common notions' (κοιναὶ ἔννοιαι).[3] There was discussion about these axioms (as Lloyd reports) and Euclid knew that his system could be used with different axioms – axioms, after all, are indemonstrable. It shows that Euclid understood that there could be

[3] Euclidian deduction and his notions of geometrical construction are different from Aristotle and modem concepts both. The same holds for the flourishing Greek geometry in the third century BCE which is also independent of Euclid (Daus, 1960; Mueller, 1974, 1981, Harari *unpublished* at the time of writing).

'non-Euclidian' geometries. He may have been a schoolmaster rather than a creative mathematician, but Euclid had a deeper understanding of the effectiveness of mathematics in describing the world than Lloyd lets on. We shall return to this apparently mysterious effectiveness.

David Pingree

Since examples abound, I move from Greece and China to India where I am more at home.

David Pingree is the author of the monumental *Census of the Exact Sciences in Sanskrit* and of numerous other publications that are indispensable aids to all serious students of the subject. In a typically solid but surprisingly technical contribution to a non-technical magazine, Pingree sets the modern period aside in no uncertain terms:

> One of the most significant things one learns from the study of the exact sciences as practiced in a number of ancient and medieval societies is that, while science has always traveled from one culture to another, each culture *before the modern period* approached the sciences it received in its own unique way and transformed them into forms compatible with its own modes of thought. Science is a product of culture, it is not a single, unified entity. Therefore, a historian of premodern scientific texts – whether they be written in Akkadian, Arabic, Chinese, Egyptian, Greek, Hebrew, Latin, Persian, Sanskrit, or any other linguistic bearer of a distinct culture – must avoid the temptation to conceive of these sciences as more or less clumsy attempts to express modern scientific ideas. They must be understood and appreciated as what their practitioners believed them to be. The historian is interested in the truthfulness of the various sciences, not in the truth or falsehood of the science itself (Pingree, 2003: 45).

I see here again a variety of strands. The italics *before the modern period* are mine because I like to underscore that the uniqueness of the modern period in the manner in which Pingree singles it out has nothing to do with the principle of modernity. The principle of modernity is not a thesis; it is an *a priori* principle. It does not possess any of the features that Pingree mentions.

I believe that Pingree would probably agree that his small Lloyd-like bow to fashion: 'science is a product of culture' does not mean the same as 'science is not a single, unified entity'. I regard it as likely that his statement is rather different from what some of the upholders of political correctness have in mind. It is not science that is a product of culture, but particular concepts of science along with particular classifications, evaluations and nomenclatures that are language-bound.

The last three sentences have me confused. Yes, we must avoid that temptation about clumsy attempts that Pingree so clearly formulates.

But he does more than merely try to understand and appreciate these ancient sciences 'as what their practitioners believe them to be'. He adopts the Needhamian yardstick of modern science when he writes about Indian mathematicians 'some were simply wrong' (p. 47) and when he refers to 'accurate values', 'discovers', 'solving significant problems' and uses phrases such as 'it had been realized in India' as he repeatedly does (pp. 46, 50–52, etc.). Making such judgments is not all there is to the principle of modernity, but it certainly is taking modern science as our yardstick.

Johannes Bronkhorst

In his reflections on Indian geometry, Bronkhorst (2001a) adopts the same yardstick as any reader can verify. He refers to some of the same 'mistakes' that were mentioned by Pingree whose assistance he acknowledges. But he is especially concerned about the absence of proofs.

I shall not elaborate here on the special context in which these investigations occur. In his 2001a article, Professor Bronkhorst has paid careful attention to several of my efforts and essays. In addition, he has invited me to participate in a conference on which more anon. I am indebted to him and have learned much from his analysis which has raised the discussion to a higher level. We agree on many basic issues and one of these is the important position that grammar occupies among the Indian sciences. But Bronkhorst's article is at the same time part of a much wider agenda which consists of several theses. He defines human rationality in a special manner which he sees exemplified by the Greeks. He feels that Indians in general possess a different kind of rationality and whenever they argue like the Greeks – which is often – he wonders whether they might not have been influenced by the latter after the period of Alexander of Macedon (who is no longer called 'the Great' by most Asians). That was an important theme in the conference 'La rationalité en Asie' organized by Bronkhorst in 1999 and edited in 2001b, and in several other publications including Bronkhorst 2001c and 2002.

Both claims, about the absence of proofs or deductions and about mistakes, have to be placed in the wider context of the history of mathematics. There have always been two ways of doing mathematics. One is geometry; it emphasizes deduction. The other is algebra; it uses computation and algorithms. The historian of science who, to my knowledge, has most recently commented on some of the ramifications of this grand division is Roddam Narasimha (2003 and *forthcoming* and see *note* 1, above).

Lloyd (2002: 65) wrote about Euclid's axiomatic-deductive demonstration that it was common, though far from universal in Greek geometry. Bronkhorst has an all-or-nothing attitude with respect to the Euclidian system. Since he has not found in India, he has concluded that Indians don't have proofs. It is true that proofs are not often given in texts, but that does not prove much in an oral culture as Bronkhorst himself admits. It is also true that Indian geometers of the Vedic period did not give axioms. But they provided deductions. Several constructions are given by making use of the theorem of Pythagoras or Baudhāyana (an example is discussed by Seidenberg 1983 followed by van der Waerden, 1983; Staal, 1999). That is not deduction from axioms but it is deduction as Seidenberg has rightly emphasized.

There is another point about proofs and deductions that is relevant to our discussion. The notion of proof, like that of exactness, is not an invariant concept. Proofs possess a history during which standards of rigor were considerably improved. In India, infinite power series were computed up to a certain point. They were recognized as approximations that could be refined indefinitely and are infinite in precisely that sense. But there was no proof of that recognition. In Europe, the infinitesimal calculus was introduced and similarly used for a long period without proper definitions or proofs. Greater precision began to be cultivated in the nineteenth century and on the European continent; the British Isles continued to lag behind. Modern mathematics and logic present a more complex story. Few people are competent to summarize it adequately and I am not one of them. But it is clear that a Euclidian derivation from axioms has been achieved only in some areas and has been proved to be impossible in others.

Bronkhorst's second claim is that Indians made mistakes. They did indeed and so did others. The list would be long so I shall confine myself to Europe and mention two examples, one at beginning and another at the end. About Plato, an eminent historian of logic has written: 'the reading of his dialogues is almost intolerable to a logician, so many elementary blunders are contained in them' (Bochenski, 1951: 17; for Aristotle, see Patzig, 1969, Register *sub voce* and for Euclid: Daus 1960). My other example concerns the proof of Fermat's Last Theorem, which was finally announced, after some 350 years, by Andrew Wiles in Cambridge in 1993. Unfortunately it contained a mistake which it took Wiles another few years to eliminate.

4. *'Rivers Flowing into the Sea...': An Empirical Hypothesis*

We have touched upon facts but not upon what is perhaps the most important thing about them: to be of use they have to be confirmed and there are degrees of confirmation. I recall that I was in Palghat and always wanted to go to Palni but cannot remember anything definite about it. I would say that the former fact, if it is a fact, is more highly confirmed than the latter. Facts are in that respect like theories about which Roger Penrose (1997: 23 ff.) tells us that General Relativity is confirmed to one part in 10^{14} and Newton's theory only to one part in 10^{7}. I cannot compute like this but believe that many geographical facts on the map of Figure 2 are much more highly confirmed than my musings about Palghat and Palni. I conclude that it is better to talk about *empirical hypotheses* than about facts.

Needham has provided us with a vision which, like all visions, stands in need of correction. Our Figure 1, which is based upon that vision, pictures an empirical hypothesis that usefully corrects widespread and persistent attitudes and prejudices. But it is too early to declare that it is highly or even widely confirmed. One of Needham's metaphors is particularly misleading. He sometimes maintains that all sciences of the past flow into the science of the future like rivers flowing into the sea. It did not always happen because some scientific insights developed in isolation, others had no impact on subsequent developments and many may have disappeared without leaving a trace behind. The case of medicine is one of these cases because much of it developed independently in the great civilizations. Even so, the Chinese were fascinated by Indian medicine and small currents of the great stream of acupuncture are beginning to be admitted by medical establishments even in the Euro-Americas. It is like the Indian Ayurveda, now part of a portfolio of alternative and complementary therapies offered alongside modern biomedicine (Wujastyk, 2003: 407). It certainly is a fact that science has always traveled from one culture to another, as Pingree put it in the passage I quoted a few pages earlier. Needham and Pingree have given innumerable examples of those travels in numerous publications, a few of them listed below in *References*.

The heliocentric case is unsettling to Needham's vision and affects another historian of science who was, in addition, a creative mathematician. Aristarchos of Samos defended the idea that the sun occupies the center of the solar system in 280 BCE; Āryabhaṭa in 476 CE; and Copernicus in 1543 CE. Copernicus knew Aristarchus: the main difference between the two is not that they pointed out something different, but that Copernicus was successful in convincing others and Aristarchus was not. Neither was Āryabhaṭa (Yano, 1980). But now a remarkable inconsistency shows up in the work of B.L. van der Waerden. His methodology was to always assume that things are related because it may display interesting interconnections. In 1969, he tried to show that Āryabhaṭa knew Aristarchus, but not that Copernicus knew Āryabhaṭa. Perhaps the latter hypothesis is more far-fetched than the former, but why did van der Waerden not pursue it?

Another counter-example to Needham pertains to mathematics and takes us closer to our theme – artificial languages. It is by now becoming more widely known that infinite power series that are expansions of π and the trigonometric functions were discovered in Kerala, southwest India, by Mādhava who flourished during 1380/1420 CE (see, e.g., Pingree, 1981: 49–51, 65–66), almost three centuries before they were discovered in Europe by Gregory, Newton and Leibniz. But these Indian discoveries were formulated in a complex form of Sanskrit and did not, as far as is presently known, flow into the science of the future.[4]

The same mathematical example led to different results in Europe where they support Needham's empirical hypothesis. In Europe, infinite power series led to the calculus and were a powerful ingredient of the scientific revolution. It is true that Newton's Latin was also obscure, but Europeans began to express knowledge with the help of algebraic notations that were easy to learn and soon spread all over the world. It explains their revolutionary impact and is in accordance with Leibniz' view, that progress in mathematics is largely due to improved notations.

[4] Results of the Mādhava school reached the Tamil country but no other places in India where mathematicians were writing in Sanskrit and could have understood them. There is circumstantial, but no direct evidence to support the idea that some of Mādhava's discoveries may have reached Europe. A group of scholars at the University of Exeter is collecting more information (The Aryabhata Group, 2000). One difficulty is that the Sanskrit of Mādhava and his followers is much more difficult than the *Arabick* that European scientists since the Renaissance were eager to learn.

Leibniz played a key role in the entire development. Throughout his life, he pursued the idea of creating a universal, philosophical language that would be an ideal language for the expression of thought which, for him, included what is now often referred to as scientific knowledge. No other mathematician has introduced as many symbols that are still in use. Directly and indirectly inspired by the algebra that had been imported via the Arabs, Leibniz' notations flowed into the sea of modern science.

5. *The Principle of Acceptance*

Grammar or linguistics is another science that fits Needham's vision. It takes us to my chief topic which is to show, that and why formal or artificial languages occur across sciences and civilizations. Its inclusion here is also in accordance with an other principle of the history of science. I refer to it as 'The Principle of Acceptance': *When studying the history of science at an earlier period or in another civilization, we should not blindly follow our own principles, concepts and categories of science, but accept those that were prevailing then and there.*

The Acceptance Principle is nothing new in the comparative study of civilizations. With regard to India, one of the essential lessons taught by Daniel Ingalls (as formulated by Pollock, 1985: 499 with special reference to Ingalls, 1965) was 'how important it is to take account of traditional categories and concepts when attempting to understand the cultural achievements of ancient India'. A simple illustration of the Acceptance Principle is the term *science* itself for it is by no means universal. Its Anglo-American use emphasizes what are sometimes called *the exact sciences*, an expression generally confined to mathematical sciences and sciences of nature. It excludes the human and social sciences, and sometimes the life sciences as well, all of them disciplines that are included in French *science*, German *Wisenschaft* or Japanese *gaku*. English usage originated in nineteenth century German philosophy (particularly Wilhelm Dilthey: Staal 1990, 1993, Chapter 29). If we go back further into European and Arab history we find that grammar, rhetoric, logic, and music were regarded as scientific disciplines at least until Newton. Music included not only acoustics and musical theory but also the art of composition (cf. Cohen, 1984). Is Bach's *Art of the Fugue* not exact and scientific even though it is also regarded as art and not clear what it is a science of?

The reader may wonder whether I am digressing but there is a reason. I must confess that I differ on this issue with David Pingree. I am not quibbling here, as I did in the previous section, with regard to some phrases in a small and perhaps inconsequential paper, but raising a question that comes to the fore in the authoritative and monumental *Census of the Exact Sciences in Sanskrit*. As Pingree tells us himself in the *Introduction* to the second volume (1971), 'the author's conception of the scope of the work has broadened'. The exact sciences have now come to include, along with mathematical astronomy and horoscopic astrology, the science of divination, geography, cosmology and other disciplines that involve the determination of the proper times for the performance of ritual acts. And yet, there is no mention of grammar which is in India the paradigm of what an *exact science* should be like.

Pingree's emphasis on the exact sciences comes from Neugebauer, who wrote mostly about Mesopotamia as does Pingree about India. Perhaps Neugebauer was right about Mesopotamia although grammatical texts existed there from around 1600 BCE, when there was concern about Sumerian becoming obsolete. I am not competent to pass judgment on whether the grammar that is discussed in these texts, and about which there exists an extensive literature, should be regarded as an exact science. As for India and Sanskrit, it has never been doubted that grammar was precisely that, which does not imply that it was at any time perfect or free from error.

I am not complaining of the factual omission of the grammatical sciences from Pingree's *magnum opus* for which there are good practical reasons. If grammatical contributions were included, the size of the *Census* might have to be more than doubled (see, e.g., Scharfe, 1977, a work that appeared in the same multi-volume *History of Indian Literature* as Pingree, 1981). But shouldn't their existence at least be *mentioned*? Isn't the neglect of linguistics a form of cultural prejudice due to the fact that Euro-American students of language embarked only recently (and not without assistance from the Indian tradition) on the scientific study of that discipline?

If we exclude the most recent-period, educated Indians have for millennia regarded it as obvious that grammar is not only 'the science of the sciences' (*śāstrāṇāṃ śāstram*), but also the most exact science. Countless texts affirm it and verse celebrate its infinite extension:

Boundless indeed is the science of language,
But life is short and obstacles are numerous.
Hence take what is good and leave what is worthless
As geese take milk from the midst of water.

To understand the most popular proverb that illustrates the brevity of grammatical rules that grammarians seek, we must know a fact about Indian culture. Many Indians prefer sons to daughters because sons perform rites that profess to take care of their parents in the afterlife. If the reader opines that such a belief is politically incorrect and/or a superstition, that is fine with me; but it explains the saying to which I want to draw attention because it illustrates the extraordinary concern for exactitude that characterizes the Indian science of language: 'grammarians rejoice over the saving of half a syllable in a grammatical statement as over the birth of a son'.[5]

Were Pāṇini and other Indian grammarians interested in brevity for its own sake? There has been much discussion of that question but Kiparsky (1991) has demonstrated that economy was Pāṇini's way of achieving generalization. Kiparsky always adheres to the principle of modernity. He does not treat Pāṇini as an exotic object of philology, but as a colleague of genius who deserves to be taken seriously and may be right or wrong.

The importance of grammar in Indian civilization has been expressed by Louis Renou in his felicitous manner: 'Adhérer a la pensée indienne, c'est d'abord penser en grammairien'.

II THE SCIENCES OF LANGUAGE AND THEIR APPLICATIONS

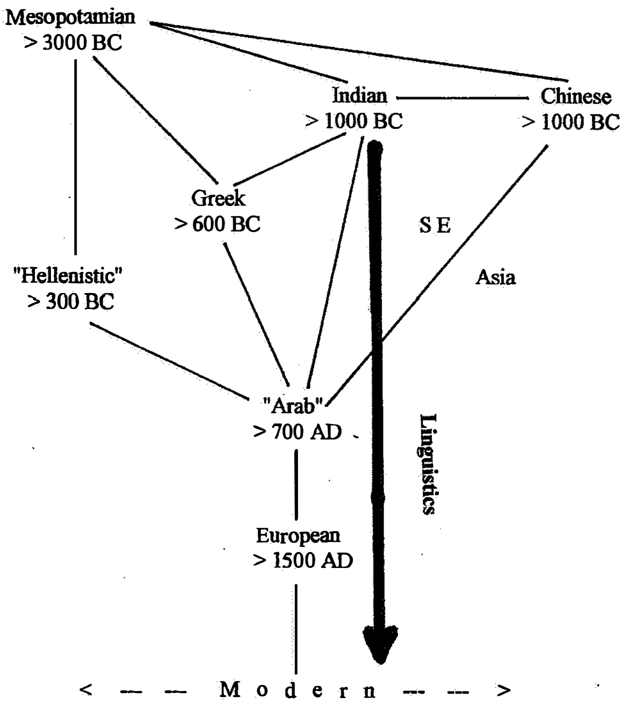
6. *Linguistics: An Apparent Tunnel*

That the chief concern of the Indian sciences has always been with human language is apparent from the earliest classifications of sciences. During the late Vedic period, around the middle of the first millennium BCE, a list is given of sciences that are called 'limbs of the Veda' (*vedāṅga*). They are the science of ritual (*kalpa*) to which geometry (*śulba*) is attached; phonetics and phonology (*śikṣā*); etymology (*nirukta*); grammar (*vyākaraṇa*); prosody (*chandas*); and astronomy/astrology (*jyotiṣā*). My translations of the names of these sciences are approximate for the boundaries of science are not the same across civilizations. It should be noted that four out of these seven deal with language. In the sequel, I shall not bother about

[5] The verse: *anantapāram kila śabdaśāstraṃ svalpaṃ tathāyur bahavaś ca vighnāḥ / sāraṃ tato grāhyam apāsya phalgu haṃsair yathā kṣīram ivāmbumadhyāt.* The proverb (which also appears as the final metarule of a grammatical text: *Paribhāṣenduśekhara* 122); *ardhamātralāghavena putrotsavaṃ manyante vaiyākaraṇāḥ.*

these subdivisions but group them loosely together under the label of linguistics or grammar.

How may we be able to embed Indian linguistics within the chart of Figure 1? It emerged again in nineteenth century Europe and so it looks as if there existed *a tunnel*, dug beneath the more clearly visible manifestations of other sciences and therefore undetected by historians of science: see Figure 3. But looks are misleading. What we have here is a construct due to the neglect not only of indigenous



Vedic (< 5th c. BCE), Pāṇini (5th c. BCE), Patañjali (150 BCE), *Kāśikā* (7th c. CE), ... , Bhaṭṭojī Dīkṣita (17th c. CE), Nāgojībhaṭṭa (18th c. CE) ...

Wilkins, Charles (1808), *A Grammar of the Sanskrita Language*, London

Bopp, Franz (1816), *Über das Conjugationssystem der Sanskritsprache, in Vergleichung mit jenem der griechischen, lateinischen, persischen und germanischen Sprachen*, Frankfurt

Thieme, Paul (1982-83), "Meaning and Form in the 'Grammar' of Pāṇini," *Studien zur Indologie und Iranistik* 8/9:3-34

Figure 3. Linguistics: an apparent tunnel.

categories but also of indigenous sources. There exists, in fact, a continuous development from the Vedic systems via Pāṇini (4th century BCE), the greatest Sanskrit grammarian, and many others including Patañjali (150 BCE) and the *Kāśikā* (7th c. CE), up to Bhaṭṭojī Dīkṣita (17th c. CE) and Nāgojībhaṭṭa (18th c. CE). The Indian tradition did not stop there, but it is then that it was picked up by Charles Wilkins whose Sanskrit grammar of 1808 is based upon Pāṇinian principles. It inspired Franz Bopp (1816) and led to the beginning of modern linguistics. Paul Thieme (1982–1983) tells the story in a few pages at the beginning of an article on Pāṇini's grammar. Indian grammar contributed not only to modern linguistics. It spread over large parts of Asia in an applied form similar to the applications of physics and other basic sciences in technology.

Indian linguistics originated among reciters who wanted to preserve their Vedic heritage and apply it in ritual. Unconcerned with meaning, they concentrated on form and incorporated a good measure of linguistic analysis that culminated in the Sanskrit grammar of Pāṇini (which includes many Vedic forms referred to as such). To understand the *European* aftermath, let us take a look at the latest Indian grammarian who adorns our brief list. Nāgojībhaṭṭa was neither a path-breaking scientist like Pāṇini, nor a slavish follower. His investigations into the Pāṇinian system were sophisticated at a forbiddingly technical level. When I wrote in the preceding paragraph that Wilkins picked up the tradition, I did not imply that he started at the point that Nāgojībhaṭṭa had reached. He was in no position to understand Nāgojī's work which European and American savants began to assimilate only much later. Like many other Sanskritists, Wilkins learned the basic Pāṇinian techniques in India from Indian *pandits* or scholars. He learned enough to be able to write a type of grammar that Europe had not seen before.

The application of pre-Pāṇinian grammatical insights from India that spread over Asia is a different story. It deserves three separate sections: one on the insights themselves, one on their spread to South, Southeast, Central and East Asia, and a third on their failure to establish themselves in the Near East and reach Europe.

7. *The Vedic System of the Sounds of Language*

One of the greatest discoveries of the Vedic reciters was the articulatory base for the production of the sounds of language insofar as they are manifest in Vedic Sanskrit. Their discovery probably started with some of the regularities displayed by the square of twenty-five consonants that is depicted in Figure 4. A reader without linguistic training may understand what is at stake by reflecting for a moment on how to produce the sounds of the first row: *k* is produced by pressing the back of the tongue against the roof of the mouth, *c* by using the front of the tongue, *t* by using its tip to touch the teeth and *p* by using the lips. In between come the sounds of retroflection, produced by bending or flexing the tongue backward so that its tip touches the spot on the palate that is marked with a capital 'Ṭ' with a dot underneath. Thus, by going from left to right in the square of sounds, we pass from the throat at the back of the mouth to the lips in the front and that holds for each of the five rows.

If we go from top to bottom along any vertical column, we move from an unaspirate to an aspirate unvoiced consonant by adding air,

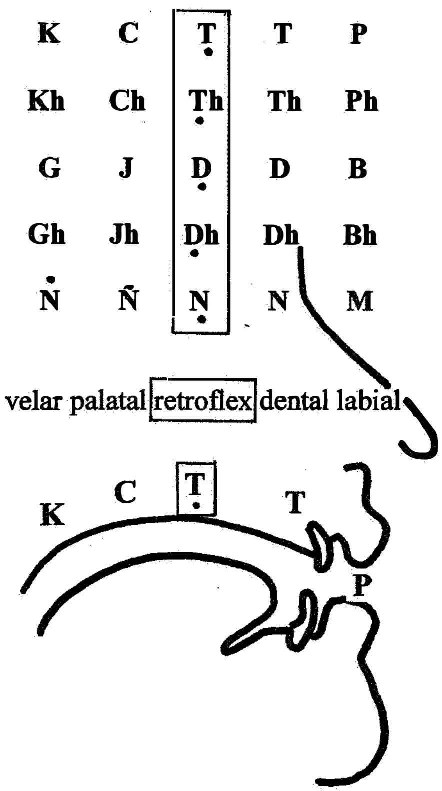


Figure 4. The Vedic system of the sounds of language.

repeating the same for its voiced relative, and reaching at the bottom the nasal of the same family which is created by passing air through the nasal cavity while positioning the tongue and lips in the same places from left to right. It must have taken time to put all these facts together and develop and extend the system by assigning to other consonants, semivowels and vowels their place and articulatory source. Pāṇini adopted a different order as we shall see.

Figure 4 depicts the sounds by letters, but the system arose in an oral tradition. For ease of pronunciation, the consonants and semivowels were followed by a short *-a*. When scripts were developed later on the basis of this system, they were generally syllabaries with a stroke, curl or other special device added to turn *ka* into *ki*, *ca* into *ci*, *ta* into *ti*, etc., another to turn *ka* into *ku*, *ca* into *cu*, *ta* into *tu*, etc., and so on for the others and similarly for the other consonants and semivowels.

8. *Indian Scripts of Asia*

Once we know and understand the natural order of the sounds of one language – a discovery that seems to have been made only once – it is not difficult to extend and adapt it to other languages that possess different sound systems. This is exactly what happened in many parts of Asia. We sometimes know about it from other sources but the primary evidence is provided by the scripts themselves. Many Indian scripts of Asia adopted the Indian *system* that developed from the Vedic system of Figure 4. Modern students of these scripts who were not familiar with the Indian system and only knew the haphazard ABC's of the languages of the Middle East or Europe have often ignored the system and paid attention exclusively to the *shapes*. A typical example is the often reprinted handbook by Hans Jensen, *Die Schrift in Vergangenheit und Gegenwart*. It denies that the Korean script derives from the Indian because *die Zeichenähnlichkeit ist überaus gering* ('the similarity between the signs is small') and must therefore be the *bewusste Erfindung eines einzelnen genialen Menschen* ('the conscious invention of a unique genius': 1969: 205). We shall see, on the contrary, that the script was developed by a committee on the basis of the Indian *system* and then expressed by a wholly original system of *shapes*. The resulting combination makes the Korean syllabary the most perfect that humans have evolved so far.

I refer to Asian scripts that adopted the Indian system as 'Indian scripts of Asia'. Many of them adopted Indian shapes as well. Others accepted some but retained or modified others inherited from their own traditions; or created new shapes, as did the Korean. Occasional uses failed to make a lasting impact. Figure 5 is a rough map that illustrates the distribution of these varieties.

The Indian system together with its shapes was adopted by almost all the scripts of South, Southeast and Central Asia. The South and Southeast Asian scripts include Kharosthi (which incorporated a few shapes from Aramaic), Brahmi, Gupta, Khotanese, Nepali, Nagari, Bengali, Gujrati, Oriya, Pallava, Grantha, Tamil, Kannada, Telugu, Malayalam, Sinhalese, Burmese, Thai, Khmer, Javanese and Balinese. The Central Asian scripts include again Kharosthi and Khotanese in addition to Tibetan and ḥPhags-pa (which was created from the Tibetan by the lama of that name for the Mongol Emperor Khubilai Khan as an international script for his Asian empire).

The organization of Chinese characters is so different from the Indian that the latter had nothing to contribute. It caused confusion

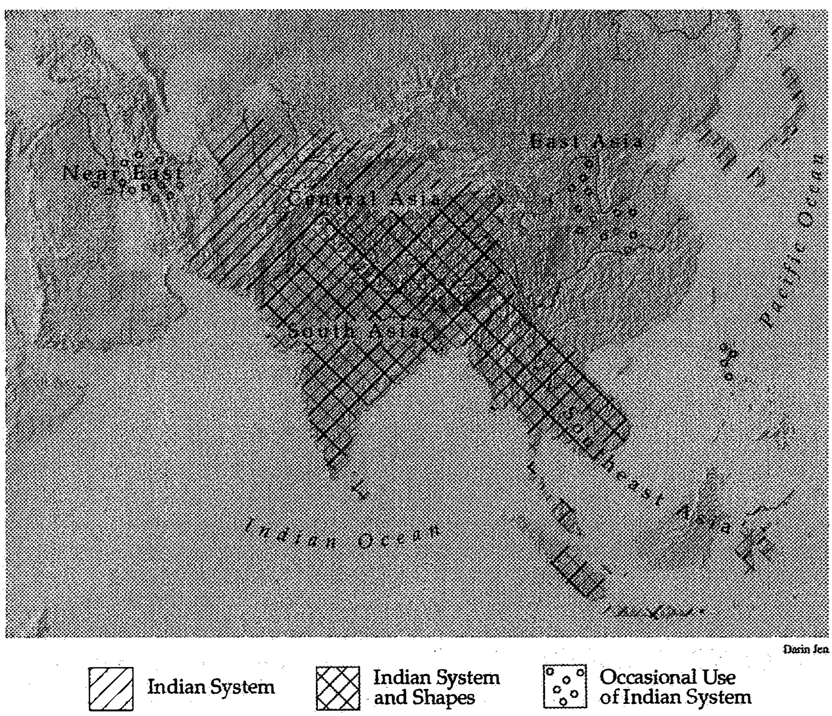


Figure 5. Indian scripts of Asia.

because Chinese Buddhists believed that each Indian shape was independent and had its own meaning, like a Chinese character. But there were a few exceptions. Hsieh Ling-yün (384–433), poet and calligrapher, assisted by Hui-jui, a Buddhist monk, composed a Sanskrit glossary in Chinese transcription in the Indian order (Zürcher, 1959, *Appendix* to Ch. IV, *note* 125 (9) on p. 412). After the ninth century, rhyme tables were composed for each tone which also adapted that organization (van Gulik, 1980: 41).

As it happens, India also exported *mantras*. They conformed to the Chinese belief because the moment one knows how to derive the pronunciation of a mantra from the way it is written, one knows all there is to know about it. That explains the success of the Siddham alphabet, an adaptation of an Indian script, which became popular in China and Japan though Sanskrit grammar itself never appealed to the Chinese. Reacting differently, Japanese and Tibetans were familiar with the Indian system and arrived at a good understanding of Sanskrit grammar after centuries of intensive study. For Tibetan, see Verhagen (1994, 2001).

In Japan, the *Hiragana* and *Katakana* syllabaries originated during the Heian period (794–1185 CE). They clearly reflect the Indian system and gradually adapted it to the sounds of Japanese (see, e.g., Wenck, 1954–1959; Lewin, 1959). In the Heian period we find, for instance, *pa pi pu pe po*, which later became *fa fi fu fe fo* and during the modern period *ha hi fu he ho*.[6] *Hiragana* is used for writing indigenous Japanese words and some Chinese loan words; *katakana* for non-Chinese (nowadays often English) loan words and technical terms such as the names of animals and plants. The shapes are simplified forms derived from Chinese characters.

In Korea, *Han–gul,* the world's most perfect script, was developed in 1444 CE by a committee of scholars appointed by the Emperor Sejong. The underlying system is Indian, as we have already seen, but the shapes of the characters are completely original and fully adapted to the sounds of Korean. They reflect the shapes of the mouth when producing these various sounds (as does the European letter 'o'). The committee report started with the basic insight, allegedly expressed by the emperor: 'The sounds of our country's language are different from those of China'.

[6] I am grateful to Professors W.J. Boot and Michio Yano who have assisted me and helped with references to relevant literature.

The study of the Indian scripts of Asia demonstrates that no system should be followed slavishly. Careful adaptation is not only more appropriate and useful. It conveys insight in the nature of the sounds and other specific properties of a language.

9. *The Arabic Alphabet*

Controversy surrounds *the Kitāb al-'Ain* or 'Book of the letter 'Ain', composed in the 8th century CE by al-Khalīl b. Ahmad, teacher of Sibawayhi, the most famous grammarian of Arabic.[7] Al-Khalīl re-arranged the letters of Arabic, starting in the back of the mouth with the letter 'Ain followed by Ḥā, Hā, Khā, Ghain, Qāf, Kāf, etc. The standard alphabet returned to the original order, bringing together letters of similar shapes that reflect an earlier stage in which there were no diacritical dots so that, for example, the shapes of the, 2nd, 3rd and 4th letter, Bā, Tā and Thā, were the same (see Figure 6).

Al-Khalīl was influenced by the grammatical schools of Kufan and Basra of which fragmentary references remain since their teachings were chiefly oral. These schools dealt with phonetic theory as well as morphology and syntax. They discussed the classification of consonants and vowels, but seem to have had nothing to say about their articulatory bases as far as I have been able to find out. Al-Khalīl may have come from Basra but he wrote his book with the assistance of a native Iranian called Laith in Khorasan, the easternmost part of Iran which is also the gateway to India (Haywood, 1960: 37).

Recent authors who deny that there was Indian influence on the *Kitāb* include several scholars writing in Arabic referred to by Danecki (1985) whom I have not been able to consult; and in addition Law (1990) and Versteegh (1993). Those who accept Indian influence include Haywood (1960), Wild (1962), Danecki (1985) and Carter (1990). Those who deny that there was any influence argue, first, that there is no evidence, meaning *texts*, for transmission of that knowledge from India to any Arab country; second, that there is no evidence for contacts between Arab and Indian scholars at the time of al-Khalīl and Sibawayhi; and third, that there are many differences between Indian theories and those of Al-Khalīl and Sibawayhi who also differed from each other in their understanding of the bases of articulation.

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[7] I am grateful to Professors Oskar von Hinüber, Richard C. Martin and Kees Versteegh who helped me with references to relevant literature.

Arabic: standard alphabet in which letters of similar shape are brought together:

ʾ	b	t	th	j	ḥ	kh	d	dh	r	z	s	sh	ṣ
ا	ب	ت	ث	ج	ح	خ	د	ذ	ر	ز	س	ش	ص
ḍ	ṭ	ẓ	ʿ	gh	f	q	k	l	m	n	w	h	y
ض	ط	ظ	ع	غ	ف	ق	ك	ل	م	ن	و	ه	ى

Sequence introduced in the *Kitāb al-ʿAyn* by al-Khalīl b. Aḥmad (718/19-91 CE), teacher of Sībawayhi, author of the first fully developed grammar of Arabic. Allegedly written in Khorasān:

ʿ	ḥ	h	kh	gh	q	k	j	sh	ḍ	ṣ	s	z	ṭ
ع	ح	ه	خ	غ	ق	ك	ج	ش	ض	ص	س	ز	ط
d	t	ẓ	dh	th	r	l	n	f	b	m	w	ʾ	y
د	ت	ظ	ذ	ث	ر	ل	ن	ف	ب	م	و	ا	ى

Figure 6. The Standard Arabic Alphabet and the Indian Alphabet of the *Kitāb al-'Ain*.

What weakens the force of the first argument is that it is widely agreed that the hypothesis of Greek influence on Arab grammar, whether directly or through Syriac intermediaries, apart from the fact that the Greeks did not have much to offer in this field, also lacks documentary evidence (e.g., Carter, 1990a: 109, 119). As for the second, around the time al-Khalīl was writing, before the ʿAbbasid period and at the beginning of Arabic science, Indian astronomy and mathematics played an important role. In 735 CE, *a Zīj al-Arkand* based on Brahmagupta's *Khaṇḍakhādyaka* was composed in Sind; in

742, a *Zīj al-Harqan was* composed but derived from Āryabhaṭa; and in the early 770's, the *Zīj al-Sindhind al-kabīr* was based on another work of Brahmagupta (Pingree, 1976, 151–69). The Indian tradition spread quickly throughout Islamic lands (Pingree 1968, 1970) and al-Khalīl, who was a great scientist with a keen analytical mind, would certainly have been familiar with its existence. As for Iran, where Indian knowledge and sciences had long been familiar and where Indian books on grammar or *vyākaraṇa*, known as *βy'krn*, had been kept along with other classics in the royal treasury since the Sassanian period (de Menasce, 1949: 2), what could al-Khalīl not have picked up there?

The third argument is the most interesting and defended in greatest detail by Vivien Law (1990) who has taken the trouble to study the Indian *Prātiśākhya* literature which is mostly available, as he rightly notes (223, *note* 2), in English translation. He has concluded that there are a great many differences of detail between Indian and Arab texts. Differences are, as we have just seen, precisely what one would expect. Arabic is very different from Sanskrit. It is far-fetched anyway to assume that Al-Khalīl studied Sanskrit treatises; but he was very versatile, wrote works on syntax, prosody and music, and it is hard to believe that he did not understand the Indian system when he heard about it, as did many others before and after him. He put the letter 'Ain, which does not occur in Sanskrit, at the beginning because its origin lies deepest in the throat. He was not averse to using experimental methods for there is a story about him putting his fingers in his mouth to find out how sounds are produced. It is likely that Indians did the same but there is no textual evidence to prove it.

Law's argument is based upon a series of closely related assumptions:

> It is virtually without precedent for a system – an abstraction, after all – to be taken over without any of the associated doctrine. In those cases where the borrowing of a linguistic system is historically tested – Romans from Greeks, Jews from Arabs, European vernacular grammarians from Romans, Japanese and Chinese from the West (to name only a few) – it has invariably been accompanied by the wholesale borrowing of concepts, terms (loanwords and calques) and even examples, as well as by frequent attempts to find a new function for redundant doctrine or to accomodate recalcitrant native phenomena within the foreign system. In short, the borrowing of isolated details along with a foreign system is historically well-attested, whereas the borrowing of a system without any of the details on which it rests is almost unknown (Law, 1990: 216).

In brief, what Law describes as 'historically well-attested' is exactly what I called 'slavish imitation'. Whether or not it applies to the examples he cites, we have seen that in most of the Indian scripts of Asia, the system was not followed slavishly with all its details but adapted to the language that adopted it *because the languages were different*. All these other adaptations strengthen the case for adaptation by al-K͟halīl. As for the differences, the nail was hit on the head almost half a century ago by Haywood (1960: 37): 'A comparison of his order with that of the Sanskrit alphabet shows sufficient broad similarity to suggest influence, yet enough divergence in detail to indicate an independent mind moulding borrowed ideas'.

That there is no written evidence for such borrowing apart from al-Khalīl's alphabet itself simply shows that the channels were oral. It is not different from many other Asian borrowings of Indian scripts or from the case of Ibn Sīnā (Avicenna) of whom it is said that he learned Greek philosophy and Euclidian geometry by himself (that is, from manuscripts or books), but picked up Indian numerals from his grocer (Nogales, 1990: 399). I conclude that in grammar as in mathematics, ideas that are part of an oral tradition may be picked up by others who do not slavishly follow texts but understand the subject.

10. *Paradoxes Surrounding the Transmission of Linguistics and Mathematics*

From the Indian point of view, the history of science does not look like the Needhamian rivers of Figure 1 or the tunnel view of Figure 3 at all. It rather looks like a tree with its roots in India and its branches spreading out over the Asian continent as depicted by the map of Figure 5. The Indian system spread orally, or to be more precise; through the ears, minds and mouths of merchants, monks and other travelers. Oral knowledge spreads more easily than written knowledge especially when it is knowledge of sounds. This brings us face to face with several paradoxes. The first is that the Indian system did not originate *in spite of* the absence of writing but *because* of it (Staal, 1989). The second is that its greatest applications were not confined to oral traditions, but pertained to the scripts of Asia, i.e., *writing*. The numerical expressions and other scientific traditions that Michio Yano studies elsewhere in this issue (141–158) are partly oral and partly written. Given the historical and conceptual priority of the study of language in India, Indians were predisposed to think of numerical expressions as language. It may explain another paradox, viz., that Indian mathematicians preferred orally based systems such as the *kaṭapayādi* in stead of making use of the numerals for which they were famous. Fortunately for the modern world, numerical expressions were adopted by the Arabs. Unfortunately, the Indian sound system did not take root in the Arab world and the modern world did not develop rational alphabets and linguistics as early and well as it did mathematics.

I mention these facts and paradoxes although they deserve much more discussion and may stand in need of qualification. They may help explain that the earliest artificial language was oral and did not arise in the context of mathematics but in linguistics.

III EARLY ARTIFICIAL LANGUAGES

11. *Pāṇini's Grammar of Sanskrit*

Pāṇini (4th century BCE) must have been familiar with what I have referred to as the *Vedic sound system,* depicted by Figure 4, but he ordered the sounds of Sanskrit differently and, from our point of view, counter-intuitively. To gain a general idea of the significance of such counterintuitive expressions we should recall that historian of science Alexandre Koyré taught that the world is based upon principles that we cannot intuitively grasp. We shall return to some of the reasons for this remarkable fact, but in the present context it is relevant to note that Koyré was thinking primarily of the physical universe and not of the study of language. Pāṇini' grammar, however, fully supports such a view. It shows that language is as mysterious as the rest of the world and illustrates that being human does not imply that everything that is human is immediately given and obvious to us. We shall see that Pāṇini needed an artificial language in order to express the counter-intuitive features of natural language. To understand his artificial expressions themselves we should take a closer look at the problems he faced.

Pāṇini used rules (*sūtra*) to express the facts of *sandhi* or 'euphonic combination'. The genre of *sūtra* is the preferred form of Sanskrit expression used in the early Indian sciences (enumerated at the outset of Section 6, Linguistic: An Apparent Tunnel): a *sūtra* is a concise formula-like expression (Renou, 1963; Staal, 1992). It is in prose, unlike, e.g., the poetic expressions used by some later Indian mathematicians (the Vedic geometers, who were contemporaries of Pāṇini, used *sūtra* prose). The subject is familiar to readers of the *Journal of Indian Philosophy,* who may also refer to an earlier and more

Sanskritic article in this *Journal* (Staal, 1995a) where the demonstration that Pāṇini created an artificial language is given in greater detail than in the slightly idealized form in which it will be presented here.

What *sūtra* rules are like will become apparent soon, but I must first explain what is *sandhi*. Fortunately, *sandhi* does not occur only in Sanskrit and so we may use illustrations from English and French. Both languages are, like Sanskrit, replete with *sandhi*, but the large majority of cases is not reflected by spelling and, therefore, easier to explain orally than in a written paper.

An example from English that is not reflected by the spelling concerns the vowel of the definite article *the* which is pronounced differently – at least in some English dialects – in its two occurrences in '*the* coconut' and '*the* apple'. The first *-e-* sound sounds like the *-u-* in *but* and is due to the fact that a consonant follows; the second sounds more like *-ea-* in *please* because a vowel follows. Both facts must be described by *sandhi* rules.

The example from French is reflected by the French spelling. If *il a* 'there is' is turned into a question, it does not become **a il?* but *a-t-il?* 'is there?' The insertion of *-t-* must be described by a rule of *sandhi*.

Sanskrit spelling always reflects the sounds of pronunciation. The reason is that the analysis of language was based upon the spoken language and was itself oral, not only among the pre-Pāṇinian grammarians but probably including Pāṇini himself (see Staal, 1986 with which von Hinüber, 1990 and Falk, 1993 are consistent). That had changed by the time of the commentator Patañjali (c. 150 BCE), but by then there existed already a sound foundation for linguistics and for a scientific organization of scripts as we have seen in Sections 8 'Indian Scripts of Asia' and 9 'The Arabic Alphabet'.

My illustration of a *sandhi* rule in Pāṇini's Sanskrit grammar describes the process of *retroflexion.* For retroflexes the reader is again referred to Figure 4 where they are marked with a dot underneath and listed within the rectangular box of the central column. Sanskrit possesses two other retroflexes which are not in the square but articulated in the same manner and one of which is also marked with a dot in the transliteration. The first is the semivowel *r*, which is more like the French or Italian than the English *r*. The second is a fricative or sibilant, written *ṣ* or, in popular transliteration 'sh' as in 'Vishnu'. The reader will be able to produce it like any other retroflex sound, by bending or flexing the tongue backward so that its tip touches the spot on the palate that is also marked with a capital 'Ṭ' with a dot underneath in Figure 4.

The process of retroflexion is complex. Modern grammars of Sanskrit devote much space to it and it is a notorious stumbling block to beginning students of Sanskrit. Pāṇini wanted to explain the reasons and express them with the help of a short and intelligible rule based upon general principles. He was able to do that though he needed to add a number of special cases and exceptions. I shall explain his rule, but simplify the complexities of the exceptions by grouping some of them together, following Pāṇinian principles though not the letter of the master. It will enable us to see in the next section and in a relatively simple manner how Pāṇini achieved his goals by introducing artificial expressions as well as an artificial language.

Pāṇini's rule (8.4.1) is simple:

After *r* and *ṣ*, *n* becomes *ṇ* in the same word. (1)

'In the same word' is added because this kind of retroflexion does not cross word boundaries. I shall omit it because it is not part of the artificial language. Many of the exceptions are captured when we add to (1) the following restriction which I shall later formulate as an artificial expression in Pāṇinian fashion. It occurs elsewhere in the grammar but Pāṇini does not use it here for reasons of his own:

unless *c*, *ṭ*, *t*, *ch*, *ṭh* or *th* interfere. (2)

Here are three examples:

A. No consonant occurs between *r* and *n* and since nothing interferes; retroflexion takes place: *kar-na* becomes *karṇa* 'ear'.
B. Consonants interfere but not the ones mentioned in (2). The famous Indian epic **Rāmāyana* refers to the 'comings-and-goings' (*āyana*) of Prince Rāma. The 'r' that causes retroflexion stands at the beginning. Several syllables intervene but do not interfere and the final 'n', therefore, becomes retroflex: *Rāmāyaṇa.*
C. *c*, one of the consonants mentioned in (2), interferes *in arcanam*, 'homage', and the retroflexion caused by the 'r' is blocked. We thus arrive at the correct form which is: *arcanam,* not *arcaṇam.*

12. *Pāṇini's Artificial Language*

Let us begin with a closer look at rule (2). In the table of Figure 4, it is not easy to single out the consonants that have to be enumerated – *c*, *ṭ*, *t*, *ch*, *ṭh* and *th* – by making use of general rules or principles. They occur in a rectangle that could be cut out from the table, but the table was orally conceived, understood and transmitted; not fit for cutting and pasting. Similarly, consonants that do *not* interfere, such as *k* and *p* in the first row, do not occur together; they occur at opposite ends. Being orally conceived, the table possessed a feature that facilitated pronunciation, recitation and recollection as we have seen: the consonants are not mentioned by themselves as *k*, *c*, *ṭ*, etc., but pronounced with a following short –*a*, i.e., as *ka*, *ca*, *ṭa*, etc. Pāṇini was aware of the fact that *ka* is the *name* of the consonant *k* and belongs to the metalanguage of grammar. The concept was familiar in the linguistic tradition from Pāṇini, who distinguished between *use* and *mention* (Brough, 1951), and Patañjali, who used the technical term *paribhāṣā* or 'metarule' when referring to rules about rules (Staal, 1975).

Various combinations of sounds are required to formulate the numerous *sandhi*- and other rules that the grammar needs. Pāṇini, perhaps with the help of assistants and students, studied and collected all these combinations and arrived at a different system for ordering the sounds of Sanskrit. It is not two-dimensional, like the square of Figure 4, but linear like natural language itself. The system includes metalinguistic markers that pick out the combinations he needed. This resulted in what were later called the *Śiva-sūtras:*

a i u Ṇ / *ṛ ḷ* K / *e o* Ṅ / *ai au* C / *ha ya va ra* Ṭ / *la* Ṇ / *ña ma ṅa ṇa na* M / *jha bha* Ñ / *gha ḍha dha* Ṣ / *ja ba ga ḍa da* Ś / *kha pha cha ṭha tha ca ṭa ta* V / *ka pa* Y / *śa ṣa sa* R / *ha* L // (3)

In this written representation of the sequence, the sounds of Sanskrit object-language have been expressed by small letters. They start with vowels and continue with semivowels and consonants, each followed by the short *a*. I have used capitals to indicate the metalinguistic markers of the metalanguage. Their use is explained by a metarule:

An initial sound joined to a final (indicatory) sound
(denotes the intervening sounds as well). (4)

Thus, *a*Ṇ refers to *a*, *i* and *u*, *a*C refers to any vowel or diphthong, *ha*L to all consonants (though *ha* occurs twice), etc. What we need is *cha*V which refers to precisely the exceptions listed in (2). It refers to consonants articulated in the central area of the mouth (see Figure 4) that interfere with retroflexion, unlike others that do not – e.g., *k* and *p* that are produced at the periphery, in the throat and by the lips.

Kiparsky (1991) has shown that the main principles of Pāṇini's grammar can be inferred from the listing of (3) and the artificial expressions that are formed from it with the help of (4).

Do expressions like *a*Ṇ, *a*C, *ha*L and *cha*V constitute what I have called 'the artificial language' of Pāṇini's grammar? They do not because an artificial language is more than a set of artificial expressions. It is a language that combines these expressions together into larger expressions like sentences or statements in accordance with syntactical rules. In that respect, artificial languages are like natural languages: they posses a *syntax*. They also posses, like natural language, a *semantics*. What is the domain of semantics of Pāṇini's grammar? The answer is easily given: it consists of the forms of the Sanskrit object-language. What is its syntax? That is our next question.

Let us begin by taking a look at the general structure of a grammatical rule such as (1). Many of these rules say that *A becomes B* or, equivalently, that *B has to be substituted for A* (both interpretations, the 'diachronic' and 'synchronic', were distinguished from each other before Pāṇini). Modern systems of linguistics sometimes express that relationship by an arrow as in: $A \rightarrow B$.

Pre-Pāṇinian grammarians had already adopted a convention: to refer to the *A* (in any of these expressions), they attached to it the ending of the Nominative case in Sanskrit; and for *B* they attached to it the ending of the Accusative case. In other words, they used the case endings of the object-language metalinguistically. They expressed that convention by a metarule.

Pāṇini introduced four metarules. The first is semi-artificial: it states that the subject of the rule, i.e., the element that is substituted, is expressed by the Nominative Case. The metalinguistic uses of three other Cases are laid down by three artificial metarules that refer to the Genitive, Locative and Ablative Cases:

The Genitive case ending is used for that in the place of which (something is substituted); (5)

when something is referred to by the Locative ending (the substitute appears) in the place of preceding element; (6)

when (something is referred to) by the Ablative ending (the substitute appears in the place) of the following. (7)

Applying these rules, Pāṇini formulates (1) as *r* and *ṣ* + Ablative ending, *n* + Genitive ending,

ṇ + Nominative ending (8)

which may be read as

After *r* and *ṣ*, in the place of *n*, substitute *ṇ*. (9)

There is no scope for applying metarule (5) but (8) is the general format of statements in the artificial language that occurs in many hundreds of the approximately 4000 rules of the grammar.

Pāṇini's artificial language, therefore, is not the language of his entire grammar. That is not different from other uses of artificial languages. Aristotle's artificial language consists of statements such as A, B, C and D that are logically related to each other; but they are generally embedded in Greek phrases translated as 'Therefore it is clear that A', 'But we may prove that it is true that B', 'Thus we have established the distinction between C and D'. Einstein's famous article of 1916 which introduced general relativity is riddled with formulas and equations but their relationships are similarly expressed through natural language as in: 'Because of the symmetry of A with respect to the indices B and C the third expression on the right vanishes provided D is an asymmetrical tensor as we shall assume...' In both cases, A, B, C and D are statements of an artificial language and the rest is Greek or German. Complete artificiality and formalization was sought in nineteenth century mathematics and, in logic, in Frege's *Begriffschrift*.

One characteristic of Pāṇini's artificial language is the linguistic zero, or rather: zeroes. It may be illustrated by constructing a fragment of a Pāṇinian grammar for English. In English, the formation of the plural may be described, to begin with, by something like (10), where P is the plural marker:

Noun + P $\rightarrow$ noun + suffix (*e*)*s* (10)

Additional rules or specifications have to be added to express that *–s* follows vowels and *–es* consonants. But there are exceptions which Pāṇini would treat with the help of zero suffixes, e.g.:

If the noun is *sheep* $\rightarrow$ s $= 0_1$ (zero suffix which causes no change)
if the noun is *man* $\rightarrow$ s $= 0_2$ (zero suffix which causes *man* to change into *men*)
if the noun is *women* $\rightarrow$ s $= 0_3$ (zero suffix which causes *woman* to change into *women*)

if the noun, is mouse $\rightarrow$ $s = 0_4$ (zero suffix which causes *mouse* to change into *mice*)

To sum up: Pāṇini's grammar makes use of *artificial expressions* such as *a*Ṇ, *a*C *ha*L and *cha*V and an *artificial language* that is constructed from these expressions in accordance with metarules adopted from Sanskrit syntax.

13. *Notes on Aristotle's Artificial Language and on Variables*

The basic expression of Aristotle's logic is: '*b* belongs to (ὑπάρχει) *a*' or '*a* is *b*', e.g., 'the horse is white'. Here 'horse' is subject and 'white' is predicate; it is therefore a predicate logic (unlike the logic of the later Stoa which is about propositions). Aristotle's logical language is not just a set of artificial expressions such as *a* or *b*, but a language consisting of statements or rules – in other words: an artificial language.

Aristotle's artificial language introduced quantification into these predicate expressions by constructing four basic forms:

'*b* belongs to all *a*' or 'all *a* is *b*' – later: *Aab*
'*b* belongs to no *a*' or 'no *a* is *b*' – later: *Eab*
'*b* belongs to some *a*' or 'some *a* is *b*' – later: *Iab*
'*b* does not belong to some *a*' or 'some *a* is not *b*' – later: *Oab*

The system was fully developed and further enriched by introducing modalities (*possible*, *necessary*, . . .). There exists a large literature about it, including attempts at formalization.

Aristotle was a contemporary of Pāṇini and added something that Pāṇini did not have. The expressions to which I have referred as *a* and *b* are *variables*, one of Aristotle's greatest inventions (Lukasiewicz, 1957: 7). Variables are similar to *pronouns* in a natural language like English: 'If anyone wants food, *he* should go to the kitchen; if *he* wants drinks, *he* should ask Johnny'. By 300 BCE, the Later Mohists in China also knew variables which neither Plato, nor early Sanskrit or Chinese or the later Chinese Buddhist logicians used (Harbsmeier, 1998: 333, 406). All these others may use 'such-and-such' for an unnamed entity, but if they use it twice, it may refer to different entities. Variables occur also in Chinese legal texts of roughly the same period as the Mohist logicians, e.g.: '*X*(*chia*) and *Y*(*i*) do not originally know each other. *X* goes on to rob *Z* (*ping*). As he arrives, *Y* also goes along to rob *Z* and speaks with *X*'.

I conclude that artificial languages appeared in human history in India, Greece and China around the same time. It is sometimes referred to as the *Achsenzeit*, 'the axial age', a period without exact boundaries or demarcations. It is not liable to a simplistic explanation, e.g., 'nomads settling down to agriculture' – a practice known to ants who do not only have queens and slaves but the slaves are engaged in cutting and collecting leaves. How does the appearance of artificial language look within the wider perspective of the development of human language? We shall try to answer that question in Section 15 'From Natural to Artificial Language' after sketching an illustration in Section 14 'From Predicates to Functions' first.

IV FROM NATURAL TO ARTIFICIAL LANGUAGE

14. *From Predicates to Functions*

We have seen how Pāṇini used the case-endings of Sanskrit metalinguistically, making their endings the most important syntactical device in his artificial language. Sanskrit is rich in cases and case-endings. Cases and case-endings occur in some languages only, but case-like structures are widespread. Aristotle constructed his artificial language by starting with a case-like structure, viz., the distinction between subject and predicate. Its expression is clear in Greek and the subject/predicate distinction seems to be a language universal, that is, it exists in all human languages. The subject–predicate structure, however, does not go very far. What can a subject–predicate structure *not* do?

A notorious example comes from one of the greatest builders of artificial expressions and languages. Leibniz tried to analyze 'Titus is bigger than Caius' as a conjunction of two subject–predicate statements: 'Titus is big in as much as Caius is small'. Historians of logic Kneale and Kneale comment: 'The strenuousness of his efforts to preserve the old theory that every statement ascribes an attribute to a subject shows his own uneasiness, and has stimulated later logicians to shake themselves free from this part of the tradition' (Kneale and Kneale, 1962: 324–325).

'The old theory' to which Kneale and Kneale refer is Aristotle's theory, not only the logical portion that refers to subject and predicate, but including the epistemological and metaphysical part that refers to subjects or substances and attributes or qualities. Both

Aristotle and his Indian counterparts in the Nyāya-Vaiśeṣika tradition started their logic with subject–predicate expressions and transformed that system into metaphysics by imagining that the world could be described by statements that ascribe attributes or qualities to subjects or substances. The Greek and Indian stock examples are, respectively, to ascribe *white* to *horse* and *weight* to *water*. A brief look at a textbook of physics, astronomy, economics, genetics, linguistics or any other science shows that it does not work.

What should Leibniz have done? A simple solution would be to replace the subject–predicate relation, which makes an assertion about one term, by another that makes an assertion about two; in other terms, replace *a monadic* by a *dyadic* relation. Let me try to make it a little more precise and illustrate how a powerful and flexible artificial expression may have developed from the subject–predicate structure of natural language. If not 'developed from', it might at least have been suggested by or is simply related to.

Let us start with 'the horse is white' or 'water is heavy'. Here a predicate b (*white* or *heavy*) is predicated of a subject a (*horse* or *water*). Let us write it in more formal terms as $b(a)$. This expression is inspired by logic where it is often written as $f(x)$ read as 'x is f'. Why not $a(b)$ or $x(f)$ or something else? Because, in these simple *monadic* cases, the entire expression or proposition predicates something of the subject. For example, *negation* of the proposition is the same as negation of the predicate and not of the subject. Aristotle explained it: 'The negation of *man is* is *man is not*, not *not-man is*'.

Following Whitehead and Russell's *Principia Mathematica*, modern logic uses a symbol for negation such as '~'. A proposition of the form $f(x)$ is negated by placing that symbol immediately in front of it, where it also occurs immediately in front of the predicate:

$$\sim (f(x)) = (\sim f(x)). \tag{11}$$

This identity entitles us to omit parentheses and write, without ambiguity '$\sim f(x)$'. Thus we have a good reason, for the particular form of '$f(x)$' itself – an illustration of the close relationship between Aristotle, modern logic and natural language.

The expression '$f(x)$' has the same form as a mathematical function:

$$y = f(x). \tag{12}$$

In mathematics, the function 'f' expresses a relation between two objects which specifies, for each x, at most one y, the *value* of $f(x)$. In the language of functions, it is easy to express what subject–predicate structures cannot, e.g. a dyadic structure:

$$f(x, y) \tag{13}$$

e.g., 'Brutus killed Caesar'; or a tryadic structure:

$$f(x, y, z) \tag{14}$$

e.g., 'the fly moves from A to B', or 'John gave the book to Mary'; or, generalized:

$$f(x_1, \ldots, x_n) \tag{15}$$

with (13) and (14) as special cases for $n = 2$ and $n = 3$.

Natural languages do not introduce infinity into the subject–predicate framework but use recursive devices such as 'and' to enumerate subjects, predicates or other expressions, e.g.:

Gopal is brilliant, far-sighted, and mean, (16)

which corresponds to

$$f(x) \;\&\; g(x) \;\&\; h(x), \tag{17}$$

or

Amir, Karin and Yuki are forgetful, (18)

which corresponds to

$$f(x) \;\&\; f(y) \;\&\; f(z). \tag{19}$$

The mathematicians who introduced expressions for functions were speakers and writers of one or more natural languages and familiar, at least implicitly, that is subconsciously, with some of their structures. In explicit terms, they were thinking of numbers or points as variables for x, y, etc. The history of some of these expressions is relatively well known. Much of it took place in the eighteenth century. Expressions such as (12) are due to Euler, 'the most successful notation builder of all times' (Boyer and Merzbach, 1991: 442). They had been used earlier without parentheses by Johannes Bernouilli I for the functor φ. The letters f, φ and ψ came in common use through the work of J.L. Lagrange. Leibniz was the first to use subscripts for functors.

The mathematical concept of function is much richer and deeper that these simple cases illustrate. Functional analysis covers a good part of mathematics and an even larger part of the vast body of mathematics that is used in physics. The successful use of such formal mathematical expressions suggests that they tell us something about the universe. Even so, these notations and expressions are related to forms of natural language and no animal without language is likely to have been able to discover or invent them.

15. *From Natural to Artificial Language*

Our remote ancestors developed natural language some 70–50,000 years ago; our more immediate predecessors developed artificial language a few thousand years ago. It explains that we are relatively well informed about the origins of artificial language in different civilizations and regions on the planet, but know little about the remote origins of language itself. The lack of balance in our information reflects our present perspective, the only perspective we possess at the stage of development we happen to have reached at the present point in time. It is a difference that may blind us to the fact that the two types of language are just two steps in the same direction. But are they?

In order to find out we should look at the differences and similarities between the two varieties. One difference seems to be that artificial languages were deliberately designed and mostly for a definite purpose: the expression of knowledge. (Pāṇini's grammar and Aristotle's logical works were systematic treatises, not teaching manuals; they became normative with the passage of time.) Natural language, on the other hand, seems to have been selected and grown in response to the need for communication, the exchange of information between senders and receivers. The main evidence for that view is that we share senders such as mouths and receivers such as ears with many other animals who use them for the purpose of communication. Human language is constrained by the structure of these organs and by a computational system in the human brain that may be different in important respects from the systems used in animal brains, which themselves vary enormously from species to species.[8]

A difference that is only apparent is that both kinds of language *look* different. Expressing it thus presupposes that both varieties are

[8] This is a large and controversial topic. Noam Chomsky is the main proponent of the idea that language is primarily for knowledge, not communication. His is a statement about function, not evolution. See, e.g., Chomsky (2002), Hauser et al. (2002) and cf. Staal (2001b). Ancient Chinese thinkers of various persuasions defined language in terms of communication (Harbsmeier, 1998: 47–48). Indian philosophers of language such as Bhartṛhari use the term *vyavahāra*, practice, usage, communication to characterize the use of ordinary language (see, e.g., Houben, 1995: 333, *note* 522).

written. Natural languages are often written, especially the important ones; but even in the present world, the majority of languages are not. None were written before the invention of writing, a few millennia ago. The same applies to artificial language: it is now invariably written, often in computer versions where 'written' itself is a metaphor; but Pāṇini's grammar was not written and the earlier Vedic treatises on language were neither. The phonology of Pāṇini's oral artificial language is the same as Sanskrit phonology. The corresponding property of the artificial languages of logic and mathematics is well-formedness.

The most significant similarity between natural and artificial languages is that both varieties consist of syntax and semantics. The most important properties of syntax are that it is able to make infinite use of finite means and that there may be more than one semantic interpretation of a syntactic expression. In natural language, the latter property is often called ambiguity, a notion that is surrounded by an aura of poetic metaphor and creativity. Artificial languages are also creative but in mathematical physics, something even more mysterious may happen: a syntactic expression may agree with empirical data though different interpretations are assigned to it, and sometimes no interpretation at all. A famous example of the latter is Planck's formula $E = h\nu$ which was in accordance with certain measurements, but seemed to make sense only much later after quantum mechanics was born (Dyson, 1988: 21; Farmelo, 2002a, b). An earlier example is imaginary and complex numbers: they do not make sense – for how can the root of -1 exist? But the universe appears to love them.

That we should be able to understand the universe is, of course, not obvious. And why should we be able to understand it in terms of our human language that was originally designed for practical communication amongst ourselves? Is artificial language really a better instrument since it was created for the express purpose of knowledge and understanding? We shall return to this question but conclude for the time being, that there are differences as well as similarities between natural and artificial languages. The paradox about artificial languages is that they are, in their original intent and ideal form, independent of natural language; but their origins and historical development have been inspired by it as we have seen. That latter fact is not surprising because it is unlikely that they fell from the sky. Is the artificial variety, then, a mere extension of (part of) the natural? Or is it something basically different?

16. *Conclusions and Questions*

First conclusion: the spread of artificial languages

Our first conclusion was that artificial languages exist across sciences and civilizations as my title claimed. We found that their earliest occurrences were not connected with mathematics as are their more recent and well known forms; but with linguistics and logic. Artificial languages have continued to flourish and have been adopted in many other sciences and disciplines.

Second conclusion: linguistics in India

Our second conclusion was that the revolutionary successes of linguistics in India were largely due to the use of an artificial language. Does the same hold for other sciences?

Third conclusion: physics and mathematics in Europe

At the end of Section 4 'Rivers Flowing into the Sea' we touched upon this question in relation to the European scientific revolution in mathematics and physics. We saw that Newton's Latin was often unclear. Actually, the formulas that are referred to as 'Newton's equations' were introduced later by Euler, Daniel Bernouilli and other mathematicians. It is true that we, today, can easily read them into Newton's words, but we do so by hindsight (Truesdell, 1968: 167). I concluded that the so-called 'European' revolution was not a product of Europe or of world history, but was triggered by the introduction of formulas and equations that were easy to understand, independent of the varieties and vagaries of natural languages, and led to a language that is universal (Staal, 1995a, 2003b, 2004). My third conclusion is that the revolution in physics and mathematics that took place in Europe was a revolution in language.

Is that what Galileo had in mind when he wrote that the universe is a book written in the language of mathematics (full translation in section 3 'The Principle of Modernity')? He did not. The idea that the universe is a book is a common *topos* throughout the Latin middle ages (Curtius, 1948), a reflection of the monotheistic religions of Judaism, Christianity and Islam, the 'religions of the book' as the Qur'an calls them. Galileo referred to such a book, but its characters are not letters: they are 'triangles, squares, circles, spheres, cones, pyramids, and other mathematical figures' (Blay, 1998:1). He was not thinking of algebra, that 'barbarous' art as Descartes called it[9] – ironically for the founder of analytical geometry which established a link between geometry and algebra. Like Newton, Galileo continued to work in the Euclidian tradition. His metaphors support the idea that the European revolution was a geometrical revolution, not inspired by the language of algebra and certainly not a revolution in language. Galileo, Newton and even Descartes failed to understand the power and impact of the algebraic language that was to be the language of the future.

Fourth conclusion: linguistics, mathematics and physics

My fourth conclusion is a generalization of the second and third: revolutionary progress in science often depends on the use of an artificial language. It applies to linguistics, logic, mathematics, physics and other sciences.

Perhaps the *positive* argument from Europe was not strong enough to establish the fourth conclusion. Its *negative* counterpart in India is all but decisive. Some of the facts that serve as steps in the argument are mentioned in Section 4 'Rivers Flowing into the Sea'. In Indian mathematics, infinite power series were discovered almost three centuries before it happened in Europe. Indian mathematics was, in this respect, as good as Newton's, but the Sanskrit was not artificial enough, it was not replaced by equations and the Indian development ground to a halt.

Indian mathematics

My next conclusion is more tentative. It is elicited by several questions. Indian grocers and other merchants combined the use of place value (already known to the Babylonians) with the Indian numerals including the zero that have been called an essential part of the development of civilization (Vogel, 1963: 42).[10] But why did Indian mathematicians use cumbrous expressions derived from linguistics

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[9] *Regulae ad Directionem Ingenii* IV:5 quoted by Gleich (2002: 209) who also refers to Newton's statement 'The Analysis of the Ancients is more simple more ingenious & more fit for a Geometer than the Algebra of the Moderns'.

[10] The idea that the zero originated in South-East Asia, at the interface of 'the eastern zone of the Indian and the southern zone of the Chinese cultures', espoused by Needham (1959: 10–12) and developed by Lam Lay Young, is not tenable for two reasons. First, the Indian evidence is earlier (middle of the sixth century: Hayashi, 2001: 774–775). Second, the South-East Asian inscriptions are Indian as is obvious from Coedès (1931) to which Needham refers. For 'the great movement of intercourse between India and South-east Asia', Needham could, in 1959, only refer to Grousset and Wales which were about to be outdated by a much more monumental and conclusive contribution: Coedès (1964), itself in due course supplemented by numerous more recent publications.

(Staal, 1995a; Yano, this volume)? Was it to add the prestige of the (2006, (
science of language to the humdrum activity of mathematical calculation? Was it because they thought of mathematics itself as a language, good for composing verse and telling stories and useful for doing sums? Was it for excluding outsiders?[11] Was it for several or all of these reasons? I tend to conclude that India's linguistic skill may have been not merely irrelevant or superfluous, but detrimental to the development of her mathematics – brilliant as it was.

The generosity of artificial language

I now return to the curious fact that equations may tell us something we did not know or understand before. It does not only apply to physics but to mathematics itself; and especially to algebra. It was formulated elegantly by d'Alembert, the leading French mathematician of the mid-eighteenth century (quoted by Boyer and Merzbach, 1991: 439): 'Algebra is generous; she often gives more than is asked of her'. Does it tell us something about the world or does it derive from the ability of language to make infinite use of finite means? Or both?

The mysterious powers of imaginary and complex numbers, which exhibit that same generosity, were expressed in similar terms by Leibniz and Roger Penrose. According to Leibniz, these numbers have an amphibian nature between being and non-being; and yet they are able to express real quantities and solve real problems (quoted in Tropfke, 1980, I: 153). Penrose (1989: 96) refers to magical properties that we had no inkling about at first and that are achieving far more than that for which they were originally designed.

That same magical property was the topic of Eugene Wigner's 1960 lecture *The unreasonable effectiveness of mathematics in the physical sciences*. Penrose (1999: 94 and elsewhere) lists it as the first mystery of quantum physics. John Barrow's book *The Universe that Discovered Itself* (2000) treats it in a long chapter entitled *Why are the laws of nature mathematical?* It contains six pages of an imaginary dialogue between a Platonist (Penrose's position) and a scientist who maintains that mathematics is a human invention. The thesis that begins to bridge the gap between the two positions occurs at the outset and is the one that is most strongly supported by the history of

[11] Not according to the *Parahita* ('benevolent') system of astronomical computation that was based upon the Āryabhaṭīya and introduced into Kerala in 683 CE. A Malayalam commentary explains the name as follows 'since this tradition of computation was accessible to all people, it acquired the name Parahita' (Sarma, 1954: vi), cf. Mahadevan and Staal (2003 and *forthcoming*).

science: *We derived most of our mathematics from Nature in the first place, so it would be rather surprising if we couldn't then describe Nature with it.*

This thesis does not explain that mathematics is a human *invention*, but supports the idea that it is a human *discovery* – not the discovery of a Platonic world separate from the physical universe, but of the structure of the universe of which we are a part; and the fact that we are in a position to discover it is due to our innate knowledge of certain properties of language. How can that be?

Let us first take a look at an example of how humans derive their mathematics from nature: the prehistory of the international Theorem of Pythagoras/Baudhāyana/the *Chou Pei Suan Ching*. It started with counting and the natural numbers, represented in many early civilizations by pebbles, dots, etc. It led to the discovery of triplets such as (3, 4, 5) with the property that $3 \times 3 + 4 \times 4 = 5 \times 5$ (in modern notation). Measurements of land led to similar findings and to the idea that the surface of a square with side 3 is a function of 3 (3×3 or 3^2 in modern notation). Half squares led to oblongs and triangles that could be measured with equal ease. Thus emerged the geometric manifestation of the Theorem leading in due time to many other new insights and results, all illustrating how discoveries go hand in hand with notations and other artificial expressions.

The artificial languages of mathematics and physics possess an inherent knowledge that may remain puzzling even if we get used to it or interpret it with the aid of intelligible models. Recognition of that puzzling fact led to a slogan: our goal is intelligibility not of the world but of *theories*. Quantum theory put an end to even that. Feynman (1964 = 1992: 129) famously declared 'Nobody understands quantum theory'. Stephen Hawking (1988, 1996: 232) elaborated: 'Even if there is only one possible unified theory, it is just a set of rules and equations'. Such puzzles about meaning are consistent with the fact, that controversies do not rage about the Schroedinger equation itself but about the correct interpretation of its meaning (Miller, 2003). Dirac expressed it as follows: 'My equation is smarter than I am' (Farmelo, 2003: xvii; Wilczek, 2003).

That intelligence resides in the language of mathematics was noted by Leibniz. Commenting on the alleged intelligence of mathematicians, he observed that when they deal with things other than mathematics they are not smarter than anyone else. He concluded that their apparent intelligence is due not to themselves but to the mathematical language that they have learnt (Mates, 1986:183). It does not only apply to artificial language. Every language user knows that we, whenever we learn a language, learn about the world (not only the world of humanity). Wilhelm von Humboldt devoted scattered portions of the ninth section of the famous *Introduction* to his large work on the Kawi Language of Java to the subject. It is an obscure piece of writing that endeavors to show, among other things, how the power of the individual is small compared to the might of language (*wie gering die Kraft des Einzelnen gegen die Macht der Sprache ist*). The topic itself is large but not nebulous and stands in need of more exploration.[12]

Final conclusions

To sum up my own endeavors. My first conclusion was that artificial languages exist across sciences and civilizations. It happened independently in different parts of the world. Why? I believe that many of the facts I have mentioned and several others suggest an explanation. We know already that the origin of human language caused a speed-up of human evolution. It was the decisive moment that led to modern humans. My summary conclusion is that artificial languages are similarly natural and a next step in the same direction. They appeared in various civilizations and led to scientific, that is, highly confirmed knowledge. The latest result to date is modern science. It shows that our tiny planet, endangered as it is by our noxious species, is a spot where the universe knows itself through language.

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[12] The idea that language is independent from human interpretations is displayed through syntax and similar to the idea that ritual is independent from meaning which I pursued elsewhere (e.g., Staal 1990, 1993).

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